Finite Projective Camera Model

Finite projective camera

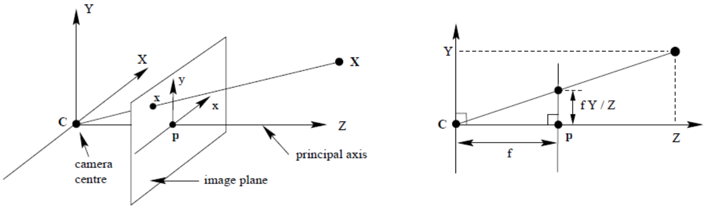


Figure: R. Hartley, A. Zisserman: Multiple View Geometry in Computer Vision

- Modeled as standard central projection onto plane.

Finite projective camera

- $\mathbf{C}$ – Camera centre, centre of projection.
- $(\mathbf{C}, X, Y)$ – Euclidean coordinate system of the camera.
- Z – Principal axis, the line through camera centre perpendicular to image plane.
- f – focal length.
- $\mathbf{p}$ – Principal point, the intersection of principal axis with image plane.
- $(\mathbf{p}, x, y)$ – Image plane coordinate system with centre at principal point.

Image of a point

- Image of point $\mathbf{X} = (X, Y, Z)$ with coordinates in the coordinate system of the camera is the point $\mathbf{x} = (fX/Z, fY/Z)$ with coordinates in the coordinate system of the image plane.
- In homogeneous coordinates this defines a homogeneous mapping

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{pmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

- In abbreviated notation $\mathbf{x} = P\mathbf{X}$, $P = K(I|\mathbf{0})$, $K = \mathrm{diag}(f, f, 1)$.
- K is called camera matrix or calibration matrix.

Camera matrix

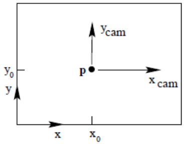


Figure: R. Hartley, A. Zisserman: Multiple View Geometry in Computer Vision

- If image plane coordinate system (x, y) is different from the coordinate system centred at principal point (x_{cam}, y_{cam}), then the callibration matrix K changes to

$$K = \begin{pmatrix} f & & x_0 \\ & f & y_0 \\ & & 1 \end{pmatrix}$$

where (x_0, y_0) are coordinates of the principal point in coordinate system (x, y).

Camera matrix

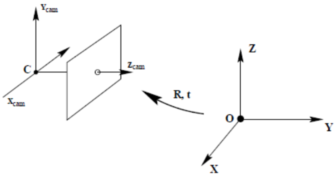


Figure: R. Hartley, A. Zisserman: Multiple View Geometry in Computer Vision

- If the euclidean world coordinate system (O, X, Y, Z) is different from the euclidean camera coordinate system $(C, X_{cam}, Y_{cam}, Z_{cam})$, the camera matrix P takes the form

$$P = KR[I| - \widetilde{\mathbf{C}}]$$

where R is orthogonal matrix, $\widetilde{\mathbf{C}}$ is camera centre with coordinates in the world coordinate system

Camera matrix

- Camera matrix $P = KR[I| - \widetilde{\mathbf{C}}]$ transforms points with coordinates in some world coordinate points into points in image plane with coordinates in image plane coordinate system.
- P has rank 3.
- The leftmost 3×3 submatrix of P is regular.

Camera matrix

- A camera with calibration matrix

$$K = \begin{pmatrix} f & s & p_x \\ & f & p_y \\ & & 1 \end{pmatrix}$$

  is called finite projective camera.
- Every 3×4 matrix with the leftmost 3×3 regular submatrix is a matrix of a finite projective camera.

Information in camera matrix – Camera centre

- For a finite projective camera with matrix $P = KR[I| - \widetilde{\mathbf{C}}]$ the camera centre has coordinates $(\widetilde{\mathbf{C}}^T, 1)^T$.
- Multiplying with camera matrix P one gets

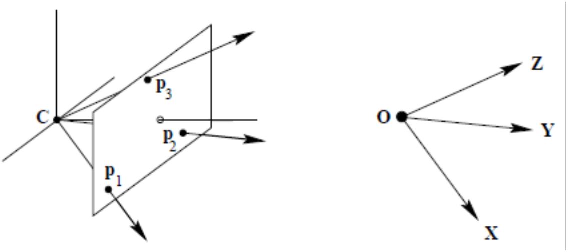
$$KR[I| - \widetilde{\mathbf{C}}] \begin{pmatrix} \widetilde{\mathbf{C}} \\ 1 \end{pmatrix} = KR\widetilde{\mathbf{C}} - KR\widetilde{\mathbf{C}} = 0$$

- From this we conclude that the coordinates of camera centre make up a vector, which solves a system of linear equations with P as the system matrix
- This is also true for a general projective matrix.

Information in camera matrix – Columns of P

- For $P = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4]$, the columns p_i represent vanishing points of world coordinate axes in the image plane.
- The point at infinity of world X axis has homogeneous coordinates $D = (1, 0, 0, 0)^T$. Its image by P is $PD = \mathbf{p}_1$.
- The image of point at infinity of a line is vanishing point of the line.
- Similarly for other coordinate axes.

Information in camera matrix – Columns of P



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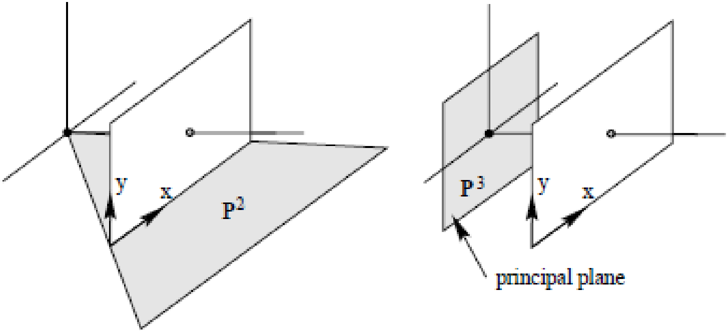
Information in camera matrix – Rows of P

- Rows of P are 4 element vectors. In general, these represent planes. Denote $\mathbf{p}^i$, $i = 1, 2, 3$ the three rows of P.
- The principal plane is a plane through camera centre parallel with image plane. Image by P of points of principal plane is a line at infinity of image plane. I.e. $P\mathbf{X} = (x, y, 0)^T$
- This gives $\mathbf{p}^{3^T} X = 0$ for all X in principal plane, i.e. $\mathbf{p}^3$ represents the principal plane.
- Similarly, rows $\mathbf{p}^1$ and $\mathbf{p}^2$ represent planes defined by the camera centre and image plane coordinate axes x and y respectively.

Information in camera matrix – Rows of P



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Principal point

- For camera matrix $P = [M|\mathbf{p}_4]$ the principal point is

$$x_0 = M\mathbf{m_3},$$

where m_3 is the third row of M.

Action of projective camera P

On lines

- Forward projection – Just an image of world point X by P.
- Back projection – Given $\mathbf{x}$ in the image plane of a camera, the back projection of $\mathbf{x}$ is a ray from centre C of the camera through $\mathbf{x}$. The point $\mathbf{X}$ of the space that has $\mathbf{x}$ as its image in image plane can be found using pseudoinverse $P^+ = P^T(PP^T)^{-1}$ of matrix P. We have $PP^+ = I$, i.e. $PP^+\mathbf{x} = \mathbf{x}$. And we can put $\mathbf{X} = P^+\mathbf{x}$.
- The ray back projected from $\mathbf{x}$ is

$$\mathbf{X}(\lambda) = P^+\mathbf{x} + \lambda C.$$